We need to maximise

$$\ln\Omega + \lambda_1 N + \lambda_2 U$$

In order to do so, we need to differentiate this. Start with

$$N = n_1 + n_2 + ... + n_i + ...$$

Differentiating with respect to n_i gives

$$\frac{\partial N}{\partial n_i} = 1.$$

Next, we differentiate

$$U = n_1\epsilon_1 + n_2\epsilon_2 + ... + n_i\epsilon_i + ...$$

This gives

$$\frac{\partial U}{\partial n_i} = \varepsilon_i.$$

Next we need to differentiate $\ln\Omega$.

Start with the formula

$$\Omega = \frac{N!}{n_1!n_2!...n_i!...}$$

Then take the logarithm,

$$\ln\Omega = \ln N! - \ln n_1! - \ln n_2! - ... - \ln n_i! - ...$$

and apply Stirling's theorem:

$$\ln\Omega = N\ln N - N - (n_1\ln n_1 - n_1) - (n_2\ln n_2 - n_2) - ... - (n_i\ln n_i - n_i) - ...$$

Differentiating, we get

$$\frac{\partial\ln\Omega}{\partial n_i} = -\ln n_i$$

Note that in this differentiation, N in Ω is to be treated as a constant. This is unlike in $\lambda_1 N$ where it is a function of n_i. The reason is because Ω is up to us to define, and it is sufficient to vary n_i.

These are what we have obtained:

$$\begin{aligned}\frac{\partial N}{\partial n_i} &= 1\\ \frac{\partial U}{\partial n_i} &= \varepsilon_i\\ \frac{\partial\ln\Omega}{\partial n_i} &= -\ln n_i\end{aligned}$$

To maximise the Lagrange function $\ln\Omega + \lambda_1 N + \lambda_2 U$, we need to differentiate this and set the derivative to zero:

$$\frac{\partial\ln\Omega}{\partial n_i} + \lambda_1\frac{\partial N}{\partial n_i} + \lambda_2\frac{\partial U}{\partial n_i} = 0$$

Substituting the above results, we get:

$$\ln n_i + \lambda_1 + \lambda_2\varepsilon_i = 0$$